

## VI. CONCLUSIONS

One of the purposes of this paper has been to offer a dynamic approach to the theoretical determination of the well-known cylindrical, or dipolar, plasma resonance frequency. Computations illustrate (see Table III) that there is agreement within 3 per cent between the dynamic and quasi-static approaches when the value of  $\beta_0 a$  does not exceed 0.25. It is shown that a small-argument expansion of the Bessel Functions in the dynamic approach yields the quasi-static solution.

The electron sheath, which exists on the outside surface of the positive column, is believed to contribute significantly to the location of the dipolar plasma resonance. Several reasons for this belief are:

1) The value of  $I_0$ , which yields resonance, is affected to a much greater degree by the temperature of the positive column than by the temperature of the mercury-pool (which governs the vapor pressure). See Fig. 2.

2) Selective cooling of a portion of the positive column affects the value of  $I_0$  of that portion to a greater degree than the other parts of the positive column. See Fig. 4 and Table V.

3) Experiment indicates that the electron sheath has a shielding effect between the plasma and the metal waveguide in the experimental setup. Compare Fig. 4 and Fig. 5, and see Table V.

4) The theoretical calculations (see Fig. 1) indicate that the ratio  $f_p/f_0$  depends upon the electron sheath density and thickness.

The correspondence between theory and experiment of the Plasma Guide Microwave Selective Coupler of Steier and Kaufman has been improved considerably by taking into account a) the temperature dependence of  $I_0$  along the axis of the discharge tube, and b) the noneffect of waveguide metal on the calculation of  $K_{\text{eff}}$  (or  $f_p$ ). However, a complete theoretical determination of  $f_p$  is not possible without information re-

garding the density and thickness of the sheath. This information is not available at this time.

## VII. ACKNOWLEDGMENT

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## Correction

Jesse J. Taub, author of "A New Technique for Multimode Power Measurement," which appeared on pages 496-505 of the November, 1962, issue of these TRANSACTIONS, has called the following to the attention of the Editor.

On page 496, the asterisk footnote should have included the following sentence. "This work was supported by the Rome Air Development Center, Griffiss Air Force Base, N. Y., under Contract No AF30(602)-2511."

On page 500, line 10 of Section IV-C, the word "even" should be "odd."